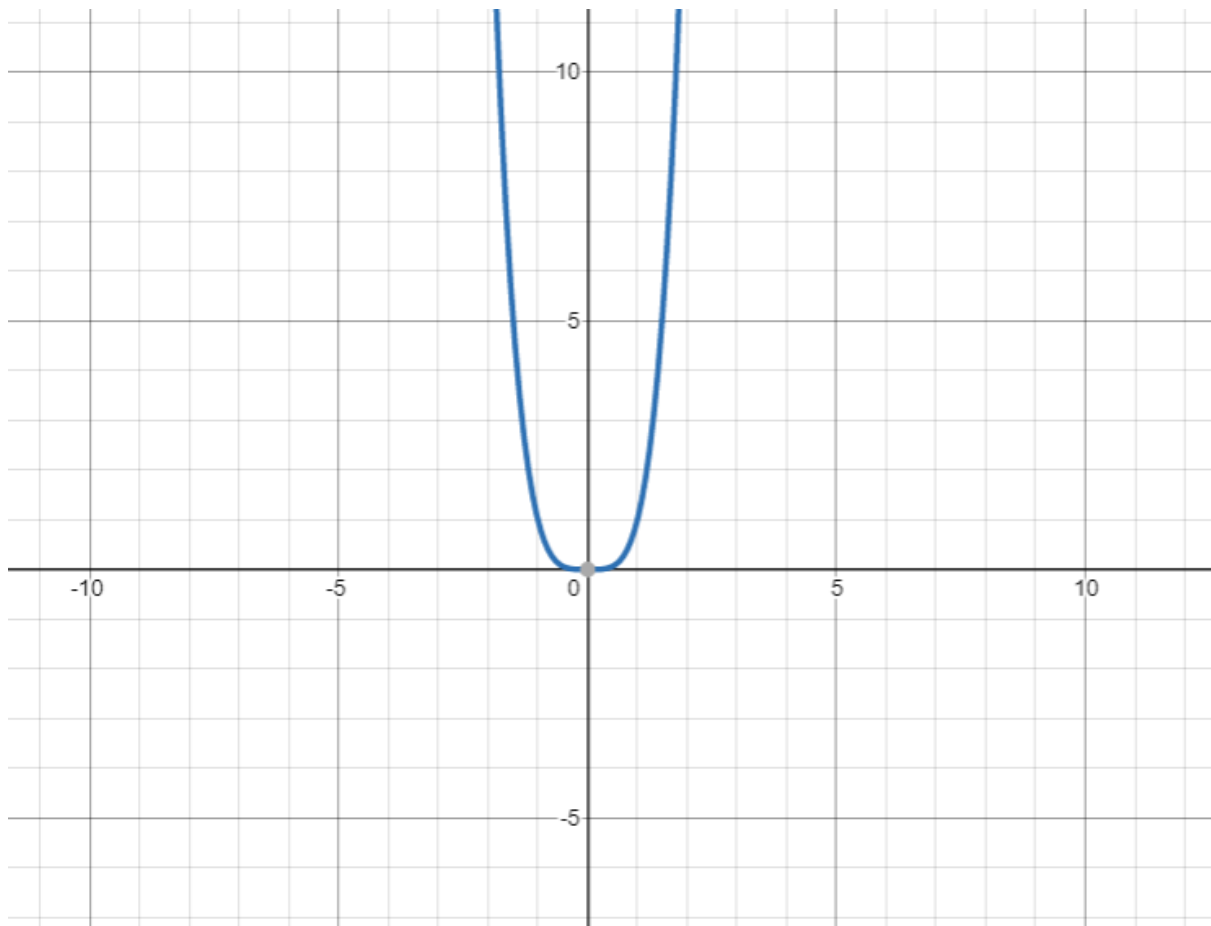


# MTH 150: Chapter 3 Questions

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(Section 3.1 Questions)

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## 1 Question 1

$$f(x)=x^4$$

As I have not handled long run behavior in quite some time, my first thought was to fact check what exactly the problem is asking me to look for. I came to realize I am to be looking for the behavior  $y$  as  $x$  approaches positive (to the right) and negative infinity (to the left), I can see that the exponent is getting larger and larger. It is clear to see that  $f(x)$  is infinity, as  $x$  approaches positive infinity. As  $x$  approaches negative infinity, the  $f(x)=x$  is 0.

(image above)

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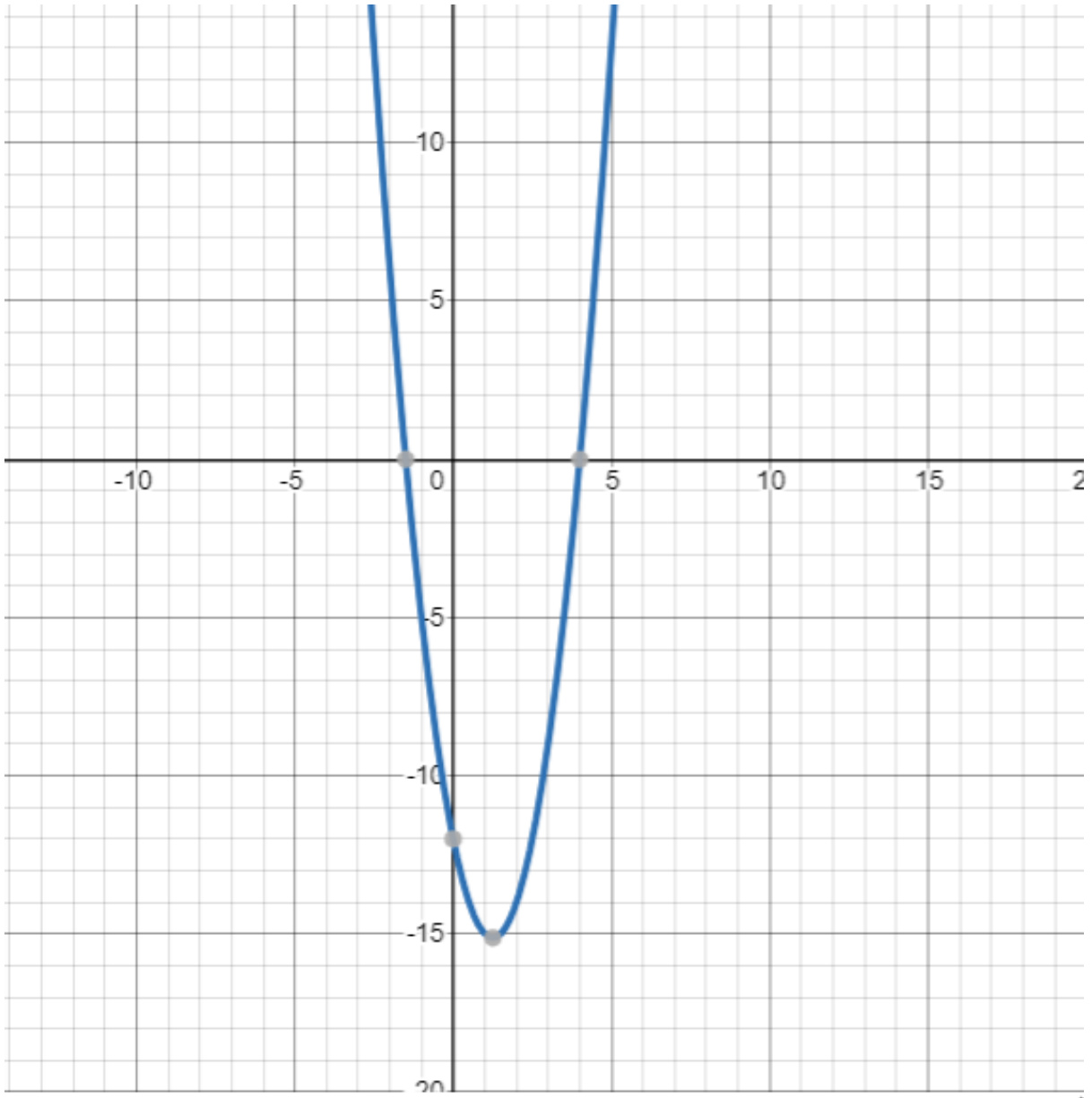
## 2 Question 15

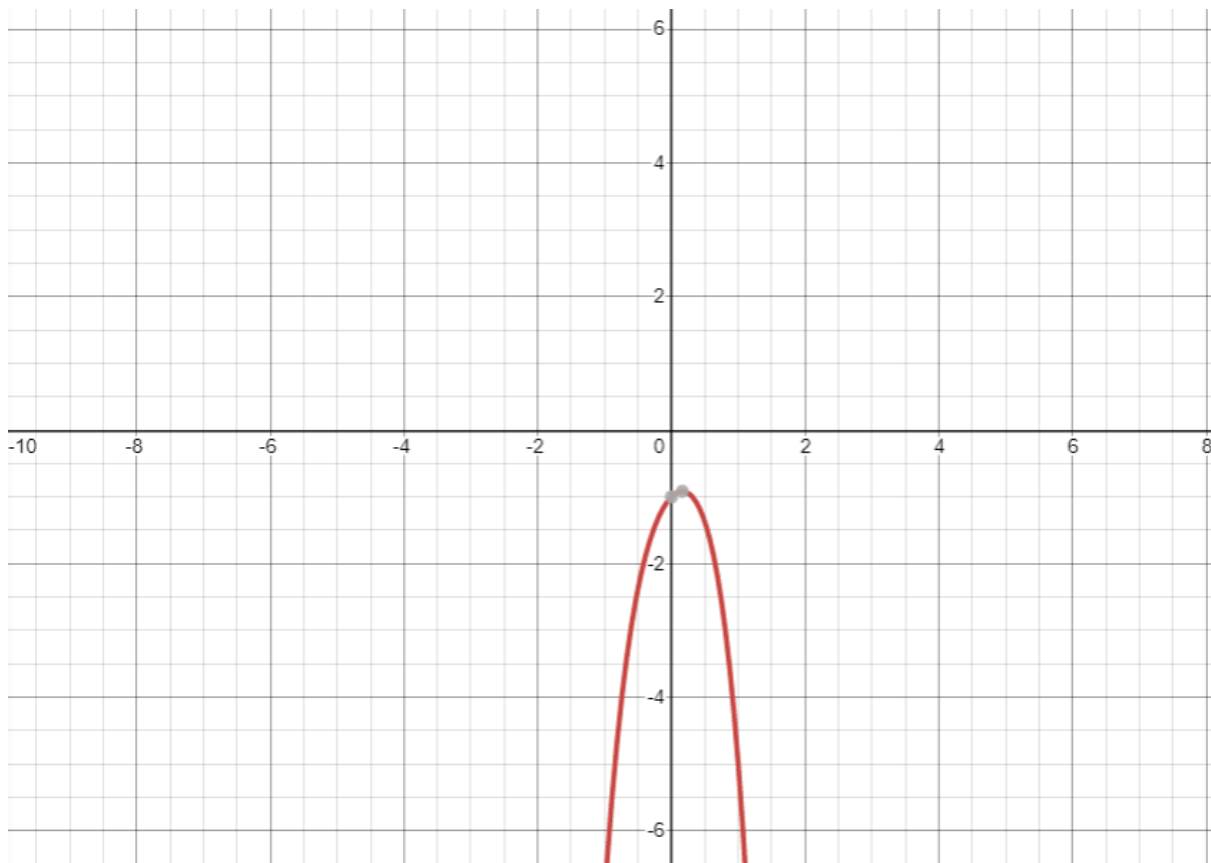
$$f(x)=(2x+3)(x-4)$$

I recognize, in order to find the leading coefficient of a function, I first looked for the coefficient with the leading power/degree. which in this case is 2 and 3. Considering the degree values of  $2x$  is 1 similar to that of  $3x$ , which is also 1.

The degree of the function is 1.

(image above)





### 3 Question 17

$$f(x) = -2x^4 - 3x^2 + x - 1$$

Similar to question 1, my first step to take in finding the long run behavior of the function above, was to utilize the desmos application, which configured the graph of the function. After having seen the behavior on the graph, as  $x$  approaches negative infinity the function values approach negative infinity.

As  $x$  approaches positive infinity, the function values approach negative infinity.

(image above)

## 4 Question 21

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Looking for the maximum number of x-intercepts and turning points for a polynomial of degree 5. I recognize that in this case: At most n horizontal intercepts. An odd degree polynomial will always have at least one. At most n-1 turning point

The polynomial (5) has a total of 4 turning points, with a maximum of 2+ horizontal intercepts. which can be suggested because of the high degree (when referring to the example out of the textbook, which used as reference for this question.

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## 5 Question 31

In order to find the horizontal and vertical intercepts of the function:  $f(t)=2(t-1)(t+2)(t-3)$

I will first analyze the behavior of the function on a graph level. Recognizing this helpful technique is the best way to arrive to an appropriate analysis of the intercepts. One can first, input the 0 as the value of t, in order to find the vertical intercept of the function. as seen below:  $f(0)=2(0-1)(0+2)(0-3)$

$$\begin{aligned} f(0) &= 2(-1)(2)(-3) \\ f(0) &= 12 \end{aligned}$$

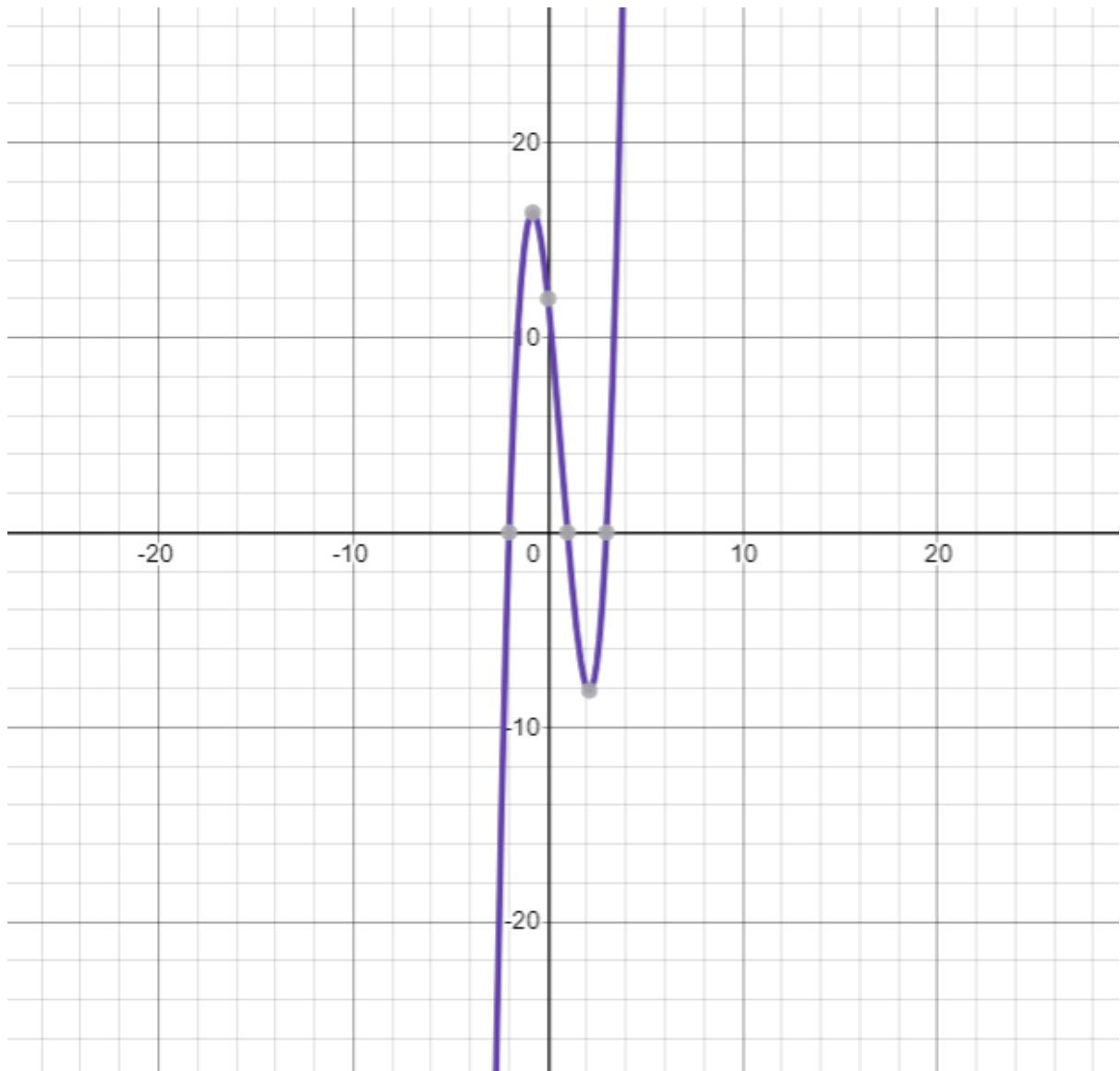
The vertical intercept is (0,12).

Whereas for the horizontal intercept, the value can be found using the various products made available in the function. And then setting the products equal to 0.  $f(t)=(t-1)$

$$\begin{aligned} 0 &= (t-1) \\ +1 &= +1 \\ t &= 1 \\ f(t) &= (t+2) \\ 0 &= (t+2) \\ -2 &= -2 \\ t &= -2 \\ f(t) &= (t-3) \\ 0 &= (t-3) \\ +3 &= +3 \\ t &= 3 \end{aligned}$$

The horizontal intercepts are: (2,0), (1,0) and (3,0).

(image above)



(Section Questions)



## 6 Question 7

$$y(x)=2x^2 + 10x + 12$$

The formula to find the x coordinate of the vertex is equal to  $\frac{-b}{2a}$

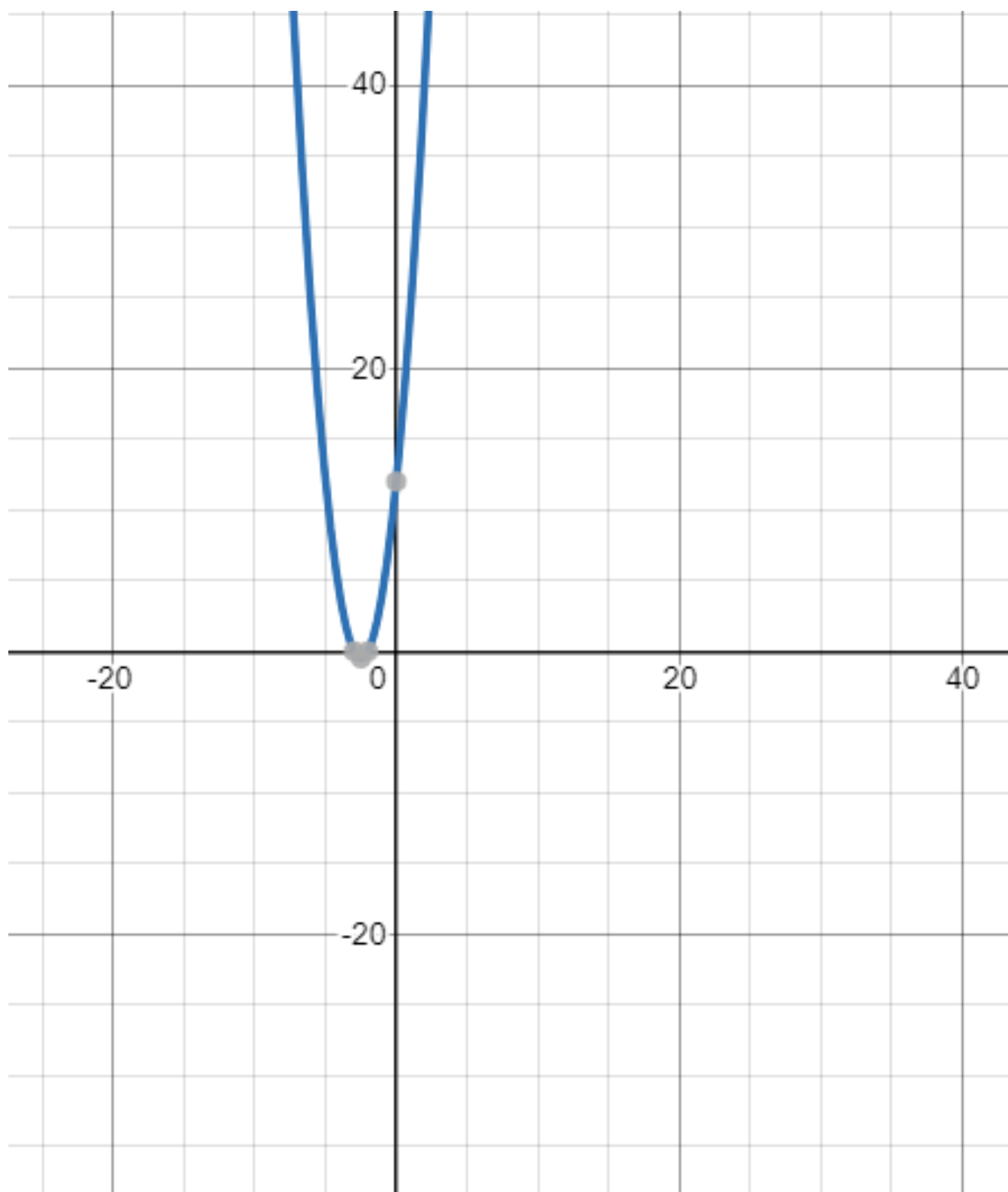
b is the coefficient on the x term, as a is the coefficient on the coefficient with the exponent value.  $\frac{-10}{2*2}$   
 $=-10/4$

$$y \text{ vertex} = 2.5 \ 1/2 \text{ vertex: } \frac{-10}{4}, \frac{1}{2}$$

For the vertical intercept, one first has to set the values of x to 0.

In order to find the horizontal and vertical intercepts of the graph, I will first plot the function using desmos to match a visual to it.  $y(0)=2(0)^2 + 10(0) + 12y = 12$  *verticalinterceptis*(0, 12)

The horizontal intercept, can also be found on the graph as (-2,0) and (-3,0)



## 7 Question 13

$$f(x) = x^2 - 12x + 32$$

Similar to the previous question, I look to use the functions points to convert it into vertex form. To do so, I personally factor into the equation- getting the simplest value of each. I will isolate the variables:

$$y(x) = 2x^2 + 10x + 12 - 12 = -12y - 12 = 2x^2 + 10x$$

$$\text{step 2: } \frac{b^2}{2} = \frac{10^2}{2} = 25$$

$$\text{Now } y - 12 + 25 = 2x^2 + 10x + 25y - 13 = 2x^2 + 10x = 25 \text{ the vertex form is: } f(x) = (x - 6)^2 - 4$$

I did struggle with this question and thought to refer to a reference online where the website broke down the instances of when it is okay to factor a function to transform it to its vertex form.

## 8 Question 19

x-intercepts (-3, 0) and (1, 0), and y intercept (0, 2)

$$(x - (-3))(x - 1) \text{ I will multiply these problems to get } x^2 - x - 3x + 3x^2 - 2x + 3$$

I used the x intercepts, yo evaluate he foundation of the quadratic value. Being that both values are expressed at the beginning of the problem, In order to incorporate the y intercept, I will multiply the quadratic function listed above in order to get the final function. Adding 2 shifts it upward, and affects the x-intercepts, too. Multiply it by a constant to make the 2 -i 3 -i 2/3

$$(2/3) * (x^2 - 2x + 3) = 2x^2/3 - 6x/3 + 3$$

## 9 Question 27

$$h(t) = -4.9t^2 + 229t + 234 \text{ a) locate original height b) when does rocket reach peak c) the splash down}$$

The initial height can be found by inputting the value of t as 0 into the equation. In doing so, I substitute 0 into the value of t and solve for h(0)

$$h(t) = -4.9t^2 + 229t + 234 \text{ h(0) = } -4.9(0)^2 + 229(0) + 234 \text{ h(0) = } 234$$

To find the maximum height of the ball, we would need to know the vertex of the quadratic.

$$h = \frac{-229}{-4.9(2)} = \frac{229}{-9.8} x = \frac{76.3}{4.96} = \frac{25.43}{1.64} =$$

The maximum height is found by the vertex

$$= (16)25.43 \frac{1}{1.64}^2 + 229 * 25.43 \frac{1}{1.64} + 234$$

$$747.52 + 5,349 + 234$$

$$\text{Vertex} = 23.36 = \text{maximum peak (height)}$$

In order to find the time the splash down occurs, I have to solve for t (time). Which allows me to solve for the time. In doing so I will set the equation equal to 0 and solve.

$$h(0) = -4.9t^2 + 229t + 234 = -4.9t^2 + 229t + 234 - 234 = -234 - 234 =$$

$$4.9(t)^2 + 229t \text{ I am personally unsure of how to solve this question, to re-}$$

familiarize myself in learning to solve in the future I will be sure to utilize the quadratic formula, which in this case would be us

## 10 Question 31

The volume of the box can be expressed as

$$V=6x^2 \text{ So if the assigned volume is } 1000, \text{ the only way to solve for } x \text{ is the equation } 6x^2 - 1000 = 0$$

Using the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  Solving this equation for  $x$  using the quadratic formula we get  $= \pm 10\sqrt{5/3}$ . Because we cannot have negative length, we are left with  $x = 10\sqrt{5/3}$  which is approximately 12.90. So the length of the cardboard is  $24.90$ . So, our piece of cardboard is  $24.90 \times 24.90 = 620$

(Section 3.3 Questions)

## 11 Question 19

To solve for  $(X-3)(X-2)^2 > 0$  each inequality

$$(X-3)=0$$

$$X=3$$

$$(X-2)=0$$

$$X=2$$

to see if these intervals are positive or negative. I recognize the interval is positive when it is part of the solution, and if it's negative it's not part of your solution. To test numbers greater than, less than, and in-between these points, You test the intervals by plugging any number  $r$  greater than 3, less than 2, or in between 2 and 3 into the inequality.

This inequality, is correct only when  $x$  is  $\geq 3$ . To prove this I input the value of  $x$  in the equation as 2, 3, or greater than 3.

$$(2-3)(3-2)^2 > 0 = -1 \text{ which is not greater than } 0$$

$$(3-3)(3-2)^2 > 0 = 0 > 0, \text{ which is true}$$

$$(6-3)(6-2)^2 > 0 \text{ Where } 48 \text{ is greater than } 0$$

Which proves that :  $(x-3)(x-2)^2 \geq 0$ , when  $x \geq 3$ .

## 12 question 21

$$(x-1)(x+2)(x-3) \geq 0$$

$$(x-1)=0$$

$$x=1$$

$$(x+2)=0$$

$$x=-2$$

$$(x-3)=0$$

$$x=3$$

Just like the previous question, my next step is determine the max. value of  $x$  so it can be applicable to the equation You test the intervals by plugging any number greater than 3, less than -2, or in between -2 and 1: To test this, I plug in for the following values:

$$\text{Any greater than } 3 \text{ (4-1)=3}$$

$$x \geq 0$$

$$(4+2)=6$$

$$x \geq 0$$

$$(4-3)=1$$

$$x \geq 0$$

Any number between -2 and 1  $(-1-1)=2$

$x \neq 0$

$(-1+2)=1$

$x \neq 0$

$(-1-3)=4$

$x \neq 0$

$(x-1)(x+2)(x-3) \neq 0$

when Any number between -2 and 1, Any greater than 3.

### 13 Question 31

Degree 3. Zeros at  $x = -2$ ,  $x = 1$ , and  $x = 3$ . Vertical intercept at  $(0, -4)$

In order to utilize what the equation provides, I begin with:  $f(x) = a(x+2)(x-1)(x-3)$ , because I know the x intercepts would be a result of the information provided in the equation. To solve for a, I plug in the values provided in the equation as the y intercept. and solve.  $-4 = (0 + 2)(0 - 1)(0 - 3)$ ,

The equation is

$$f(x) = \frac{2}{3}(x+2)(x-1)(x-3)$$

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### 14 Question 32

Degree 3. Zeros at  $x = -5$ ,  $x = -2$ , and  $x = 1$ . Vertical intercept at  $(0, 6)$

$(x+5)(x+2)(x-1) = 0$  I will input the y intercept in order to solve a in the quadratic formula

$$(0+5)(0+2)(0-1) = 9 \quad 5+2-1=9 \quad 6=9$$

divide both sides by  $3 \frac{2}{3}$

The expanded equation is:  $f(x) = \frac{2}{3}(0+5)(0+2)(0-1)$

### 15 Question 51

$y = 5 - x^2$  the area of a rectangle is  $A = xy$   $A = 2X(5 - X^2) = 10x - 2x^3$  Where the y value will be a maximum area. In doing so, by dividing the y value by the x - value gives me the height of the rectangle.

The maximum is at

so in dividing the y value by the x we get a base of.

the base is 2.58, with a height of 6.67

I struggle with this equation particularly because of a lack of depiction with an image. Having had a real description of the missing points, would have provided me a visual to pair with this function.

(Section 3.4. Questions)

### 16 Question 21

$$x^3 - 6x^2 + 11x - 6$$

$c=1$

I already knew that if the polynomial factors it reveals the roots. If  $p(c) = 0$ , then the remainder theorem tells us that if  $p$  is divided by  $x - c$ , then the remainder will be zero, which means  $x - c$  is a factor of  $p$ . If we expand the numerator and denominator of our function we get that they both have degree 2  $y = 7 \cdot \frac{(x-4)(x+6)}{(x+4)(x+5)} =$

## 17 Question 22

(Section 3.5. Questions)

## 18 Question 1

$$f(x) = x^3 - 2x^2 - 5x + 6$$

In recognizing the solutions of the equation or polynomial that cannot be represented accurately on the graph are called complex zeros. I begin to solve the equation:  $y = (x-4)^2$

## 19 Question 11

$$f(x) = x^3 - 2x^2 - 5x + 6 \text{ Find the real zeroes}$$

Similar to the problems above, the real zeroes are able to be found using Cauchy's bound.  $y = (x+8) = 2$   
 $xy + 8y = 2$  (Section 3.6. Questions)

## 20 Question 5

$$\frac{2 + \sqrt{12}}{2}$$

To simplify to a complex number, I first recognize, in order to do so, To add or subtract complex numbers, we simply add the like terms, combining the real parts and combining the imaginary parts

$$\frac{2 + \sqrt{12}\sqrt{-1}i}{2}$$

$$\frac{2 + 3.5\sqrt{-1}}{2}$$

divide the like term of 2 in both the numerator and denominator Leaving the imaginary number  $i$  and the square root of 12 which is 3.5

## 21 Question 19

Simplify to a complex

$$\frac{3 + 4i}{2}$$

$$\frac{3 + 4i}{2}$$

\*

$$\frac{-2}{-2}$$

$$\begin{aligned}
 &= \\
 & \frac{(3+4i)(-2)}{(2)(-2)} \quad \begin{array}{l} \text{The numerator} \\ \text{The denominator} \end{array} \\
 &= \frac{-6-8i}{-4}
 \end{aligned}$$

$$\frac{6-8i}{-4}$$

## 22 Question 25

Find all of the zeros of the polynomial  $f(x)=x^2 + 4x + 13$

$$\begin{aligned}
 x &= 2 + /- \\
 & \frac{\sqrt{(-2)^2 - 4(1)(13)}}{2} * 1 \\
 &= \\
 & 2 + /- \\
 & \sqrt{-48} = 2 + /-
 \end{aligned}$$

7i 2=(Section3.7.Questions)

## 23 Question 5

$$\begin{aligned}
 p(x) &= \\
 & \frac{2x-3}{x+4}
 \end{aligned}$$

.Plug  $y=0$  into the equation and solve the resulting equation  $0=p(x)=$

$$\frac{2x-3}{x+4}$$

for x. Horizontal intercept: 0) there is no horizontal intercept because when  $f(x)=0$ , there is no solution for x

Plug  $x=0$  into the equation and solve the resulting equation  $y=-3/4$

Vertical intercept: (0,-1)

Vertical asymptote:  $x=2$ , The vertical asymptote gives a 0 in the denominator, which is undefined, there is only an x in the denominator, so as  $x \rightarrow 2$ ,  $f(x) \rightarrow \infty$ . Horizontal asymptote:  $y=0$ ,

## 24 Question 19

an equation for a rational function.

\*Vertical asymptotes at  $x=5$  and  $x=-5$  x intercepts at (2,0) and (1, 0) y intercept at (0,4) Since f has a vertical asymptote  $x=-5$ , and  $x=5$  then the denominator of the rational function contains the term  $(x+5)$  and  $(x-5)$ .

Function 5 has the form  $f(x)=g(x)$

$$\frac{g(x)}{(x+5) * (x-5)}$$

The equation is:

$$\frac{(2,0) * (1,0)}{(x+5) * (x-5)}$$

## 25 Question 39

$$f(x) =$$

$$\frac{3x^2 + 4x}{(x + 2)}$$

You can find the equation of the oblique asymptote by dividing the numerator of the function rule by the denominator and using the first two terms in the quotient in the equation of the line that is the asymptote

I am unsure of how exactly to solve this equation due to my lack of knowledge on slant asymptotes. In order to recognize how this function operates I have watched reference videos and solves example problems.

## 26 Question 45

a beaker containing 20 mL of a solution containing 20

when adding water to dilute this, and  $n$  as a variable for water the next step is to create the new equation a)  $20\text{mL} = 20$

b) with 10mL of water added the new concentration is

$$20\text{mL} = 20 \text{ The concentration is } 2\text{mL}$$

C)  $20\text{mL} = 4$  the water that would be necessary for the equation is 52ml

d) the physical significance of  $n$  approaching positive infinity means the solution will be diluted while still meeting the means of the 20mL beaker,  
(Section 3.8. Questions)

## 27 Question 1

$$f(x) = (x-4)^2 \text{ (4, positive infinity)}$$

(1-1 non decreasing):

$$y = (4-x)^2 \text{ I will first begin by switching the placeholder of } x \text{ and } y \text{ in the problem, to do so I will get : } x = \frac{(8-y)^2}{4} \text{ I will begin solving for the value of } y \text{ I take the square root of both sides to get } + / - \sqrt{(x)} = 4 - y$$

$$\text{I am left with two equations } (+) \sqrt{(x)} = 4 - y \text{ } (-) \sqrt{(x)} = 4 - y$$

add  $y$  to both sides of the first equation and subtract  $\sqrt{x}$  from both sides of the first equation to get :

$$y = 8 - \sqrt{x} \quad y = 8 + \sqrt{x}$$

The inverse equations on the same domain are listed above when  $x \geq 4$ , then if  $f(x) = (x, y)$ , then

$$f^{-1}(x) = (y, x)$$

so take a value of  $x$  greater than 4, say 8, and solve for  $y =$

$$(4-x)^2 \cdot y = 8 + \sqrt{x} \text{ is your inverse function to the part of } y = (8-x)^2 \text{ that is increasing.}$$

## 28 Question 7

$y = 9 + \sqrt{4x - 4}$  Much like the previous equation, I will replace the variables  $x$  and  $y$  with each other of the above function.

$x = 9 + \sqrt{4y - 4}$  Solving for the variable  $y$ , I first subtract the integer 9 from both sides

$$x - 9 + \sqrt{4y - 4} - 4x - 9 = 9 + \sqrt{4y - 4} - 9x - 9 = \sqrt{4y - 4}$$

I will square both sides of the equation  $(x-9)^2 = \sqrt{4y-4}^2 (x-9)^2 = \sqrt{4y-4}$



$$(x-9)^2 = 4y - 4$$

I will add 4 to both sides of the equation

$$(x-9)^2 + 4 = 4y - 4 + 4(x-9)^2 + 4 = 4y$$

I will divide both sides by 4  $\frac{(x-9)^2+4}{4} = \frac{4y}{4}$

Thus, the inverse function is  $f^{-1}(x) = (x-9)^2 + 4$

## 29 Question 17

$$V = \sqrt{20L}$$

In order for a police officer to determine how fast a car was moving, that skid across asphalt 215 feet. I will input the information provided by the problem (feet) and solve for L

$$V = \sqrt{20(215)} V = \sqrt{4300} V = 65.6 \text{ mph}$$

The estimated speed of the vehicle is 65.6 mph

## 30 Question 21

Impose a coordinate system with the origin at the bottom of the ditch. Then the parabola will be in the form

$y = ax^2$  the points on either side at the top of the ditch are  $(-10, 10)$  and  $(10, 10)$ . Plugging either into the general form and solving for

0.1. To find the x -

coordinates where the water meets the edge of the ditch, plug 5 into  $y = 0.1x^2$ . Solving for x gives approximately 7.07. Note that